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Pareto Optimization for Uplink NOMA Power Control

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***Abstract*—Game theory has been employed in the determination of optimum power levels of nodes for *uplink* power domain non-orthogonal multiple access (NOMA). In this paper, a novel closed form utility function with an optimized penalty-to-reward parameter ratio is derived whose outcome gives Pareto optimum power levels. The dynamically determined power levels with this utility function have significant advantage over the static equally allocated power levels in terms of the bit error probability in the practical case of imperfect successive interference cancellation (SIC) at the receiver.**

***Index Terms*—Game theory, Pareto optimum, power domain NOMA.**

I. Introduction

Spectrum efficiency is of great importance for variety of users over a limited bandwidth in the Internet of Things (IoT) segment of 5G networks. Non-orthogonal multiple access (NOMA) is a promising multiple access technique that allows better spectrum efficiency [1]. Among different NOMA techniques, power domain NOMA has been standardized based on the superposition of different users' power at the base station and successive interference cancellation (SIC) at the receiver [2]. Assigning the optimum power levels for each user has a key role to enhance the system performance of NOMA. Although this might be readily done by the base station for the *downlink*, it is quite challenging for the *uplink* in which either each user must have complete knowledge of the other users' channels, as well as its own channel, or a central coordinator adjusts the power levels of all users globally both of which are neither practical and scalable. Similar to *downlink* NOMA, multiple users transmit at the same time and frequency with different power levels and SIC is applied at the receiver to decode the user signals in the *uplink* power domain NOMA. In this paper, power domain NOMA is studied in which each user will locally and selfishly determine its power for an *uplink* network topology, so that scalable low complexity resource allocation can be obtained, which is highly desirable for the IoT segment of 5G networks.

The key design requirement in power domain NOMA is to decode all the user signals with an acceptable error probability at the receiver depending on the power allocation among users. In the literature, the studies for *uplink* NOMA are mainly centered on static fixed power allocations, e.g., [3], as stated in [4] wherein they propose a dynamic power allocation for *uplink* NOMA based on mixed integer non-linear programming. However, mixed integer non-linear programming does not possess the scalability that is desired for 5G network design. Moreover, centralized power control schemes for *uplink* NOMA, i.e., assigning the powers for each node through iterative water filling [5] or less complex suboptimal approaches [6] is not feasible when there is a large number of IoT devices and are not considered in this paper.

In this paper, distributed game theoretical power allocation is studied for *uplink* NOMA in which each user locally and selfishly determines its power. In this regard, we seek a fair, efficient optimized utility function for a NOMA power control game, where the outputs of the game give the optimum power levels. Our utility function model is composed of a reward and penalty function based on the canonical utility function in [7] and is optimized based on the optimum power levels for NOMA found in [8]. The optimized utility function gives the Pareto optimum power levels. Although game theory has been employed extensively for power allocations in CDMA networks [9], we incorporate this methodology for *uplink* power domain NOMA. Furthermore, the previous power control game studies for CDMA networks did not consider optimizing the reward and penalty parameters [9].

The contributions of this paper are the following. First, we demonstrate that there is a unique Nash equilibrium for a general power domain NOMA model extending the Nash equilibrium for the canonical utility function that was previously introduced for CDMA [7]. Second, we determine a closed form rule between the penalty and reward parameters of the utility function to obtain Pareto efficient results. Third, the power levels determined with the optimized utility function is determined for *uplink* power domain NOMA, and its efficiency is shown in terms of bit error probability in case of imperfect SIC detection at the receiver. It is further shown that imperfect SIC does not significantly affect the error probability of other users' symbols when their powers are adjusted according to the proposed game theoretical approach.

The structure of this paper is as follows. In Section II, a game theoretical power allocation scheme for *uplink* power domain NOMA is discussed and the acquired power levels are numerically evaluated in Section III. The paper ends with the concluding remarks in Section IV.

II. Power Control Game for NOMA

The NOMA model consists of N nodes that will send their symbols to an access point (AP) at the same time and frequency with different power levels and the AP decodes all the symbols using SIC at the receiver as shown in Fig. 1. Therefore, the transmission power levels of nodes will directly affect the efficiency of decoding. This scenario can be modeled as a game played among the nodes in the *uplink* network. The game is the decision process to determine the power that each node will transmit. If they all transmit at full power, severe interference will be created and none of the transmissions will be successful where a successful transmission is defined as having a desired level of signal-to-interference-plus-noise ratio (SINR) at the receiver. On the other hand, it is not a reasonable strategy for each node to transmit at low power, because they are not cooperating and do not know how the others will behave. As a result, it is essential to find the power levels based on the Nash equilibrium. There can be many Nash Equilibrium points, and changing the game parameters will result in different power values for each node. Therefore, it is important to determine the Pareto optimum solution [9], which gives the optimum power levels of the nodes.

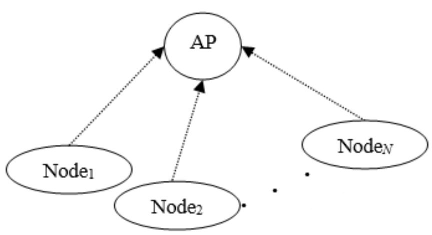

Fig. 1. System model

The game G can be formulated as $G = (N, \{p_i\}, \{J_i(.)\})$ where $N = \{1, 2, \cdots, N\}$, p_i is the strategy set represents the power levels, which is $p_i \in D_i = (0, p_{max})$, and $J_i(.)$ is the utility or cost function as $J_i(.) : D \to R$ where $D = D_1 \times D_2 \times \cdots \times D_N$ and $\times$ denotes for Cartesian product, and R is the set of real numbers. The cost function $J_i(.)$ is minimized by the i^{th} user.

The choice of the utility function deeply affects whether the solution is Pareto optimum or not. Therefore, it is critical to use an efficient and fair utility function. One of the biggest challenges in game theory is to select a proper utility function. It is important to emphasize that selecting a different utility function will give a different solution, or Nash equilibrium, if it exists. Moreover, a utility function can have a unique Nash equilibrium, whereas another utility function may not even have a Nash equilibrium. Therefore, determining a proper utility function is among the most challenging aspects of applying game theory to the optimization of wireless networks.

Many different utility functions are employed for the power control of nodes to reduce the overall interference in the network, see [10] and references therein. As noted earlier, a canonical utility function that is used for power control in an *uplink* CDMA network within a single cell is composed of a utility function composed of the reward and penalty parts [7]. The price paid for unit power is Lagrange multiplied by a parameter that constitutes the penalty function while the capacity benefit that each node gains due to its transmission power is multiplied by another parameter representing the reward function [7]. However, the relation between the penalty and reward parameters is missing in [7], and their analyses and algorithms require processing gain, which is not the case for power domain NOMA. That is, we generalize the results of [7] to the power domain NOMA.

The utility function model that will be used for this power control game within power domain NOMA is

$$J(p_i, \mathbf{p}_{-i}) = P(p_i, \mathbf{p}_{-i}) - R(p_i, \mathbf{p}_{-i}) \tag{1}$$

where $P(p_i, \mathbf{p}_{-i})$ and $R(p_i, \mathbf{p}_{-i})$ are the penalty and reward functions respectively, and p_i is the transmission power of the i^{th} user, while $\mathbf{p}_{-i}$ is the power of all users except the i^{th} user. The power control game is expressed as

$$argmin_{p_i} J(p_i, \mathbf{p}_{-i}), \forall p_i > 0. \tag{2}$$

More rigorously, the utility function is expressed as

$$J(p_i, \mathbf{p}_{-i}) = \alpha_i p_i - \beta_i \log_2(1 + \gamma_i) \tag{3}$$

where α_i and β_i represent the penalty parameter and reward parameter respectively in (3), and

$$\gamma_i = \frac{h_i p_i}{\sum_{j \neq i} h_j p_j + \sigma^2}. \tag{4}$$

where h_i is the channel gain for i^{th} user and σ^2 is the variance of the noise.

Lemma II.1. *There is a unique Nash equilibrium for the NOMA power control game assuming that there are N of active nodes whose power is greater than 0.*

Proof. Taking the derivative of (3) with respect to p_i produces

$$\frac{\partial J(p_i, \mathbf{p}_{-i})}{\partial p_i} = \alpha_i - \frac{\beta_i h_i}{\sum_{j \neq i} h_j p_j + h_i p_i + \sigma^2} \tag{5}$$

and its second derivative is

$$\frac{\partial^2 J(p_i, \mathbf{p}_{-i})}{\partial p_i^2} > 0. \tag{6}$$

Then, equating (5) to 0 with the assumption that N users are active, and writing this in matrix notation yields

$$\begin{bmatrix} 1 & h_2/h_1 & \cdots & h_N/h_1 \\ h_1/h_2 & 1 & \cdots & h_N/h_1 \\ \vdots & \vdots & \vdots & \vdots \\ h_1/h_N & h_2/h_N & \cdots & 1 \end{bmatrix} \begin{bmatrix} p_1^* \\ p_2^* \\ \vdots \\ p_N^* \end{bmatrix} = \begin{bmatrix} \beta_1/\alpha_1 - \sigma_1^2 \\ \beta_2/\alpha_2 - \sigma_2^2 \\ \vdots \\ \beta_N/\alpha_N - \sigma_N^2 \end{bmatrix} \tag{7}$$

where $\sigma_i^2 = \sigma^2/h_i$.

As proven in [7], the matrix becomes full rank provided that $N > 1$ and there exists a unique Nash equilibrium depending on the fact that the matrix is invertible and hence the set of optimum power levels, p_i^*, has a unique value. Since a multiple access scheme is considered in this paper, N must be greater than 1, so that the matrix is full rank and there is a unique Nash equilibrium. □

Lemma II.2. *There is a closed form rule between the penalty and reward parameters that makes the Nash equilibrium Pareto optimum, which is equal to*

$$\frac{\beta_i}{\alpha_i} = \frac{\sigma^2\gamma_r \sum_{i=1}^{N}(1+\gamma_r)^{i-1} + \sigma^2}{h_i} \tag{8}$$

where γ_r is the predetermined SINR level at the receiver to make a successful decoding.

Proof. We first derive a relation between penalty and reward parameters assuming there are 2 nodes without any loss of generality and then generalize this result to N nodes. It was previously shown that the optimum power levels for NOMA-SIC becomes [8]

$$p_i = \frac{\sigma^2}{h_i}\gamma_r(1+\gamma_r)^{i-1}, i = 1, 2, \cdots, N \tag{9}$$

such that each user has equal bit error rates (BER) at the receiver assuming perfect SIC at the receiver. Using (9) in (7) for 2 users results in

$$\frac{\beta_i}{\alpha_i} = \frac{\sigma^2\gamma_r + \sigma^2\gamma_r(1+\gamma_r) + \sigma^2}{h_i} \tag{10}$$

which can be written for 3 users as

$$\frac{\beta_i}{\alpha_i} = \frac{\sigma^2\gamma_r + \sigma^2\gamma_r(1+\gamma_r) + \sigma^2\gamma_r(1+\gamma_r)^2 + \sigma^2}{h_i}. \tag{11}$$

Generalizing (10) and (11) to N users gives (8), which completes the proof. □

Although the Nash equilibrium can take different values based on the different penalty and reward parameters, any penalty-price relation that satisfies (8) have unique Pareto optimum Nash equilibrium.

Corollary II.3. *The game $G = (N, \{p_i\}, \{J_i(.)\})$ with the utility function given in (3) that has a relation between penalty and reward parameter as in (8) has a unique Pareto optimum Nash equilibrium which is equal to (9).*

Proof. Following Lemma II.1 and II.2 gives Corollary II.3. □

Many important results can be deduced from this formula and are summarized below:

- There is a closed form rule between the penalty and reward parameters depending on the noise power, predetermined SINR level, and channel estimation.
- The power levels determined using the power control game are not the optimum solution without prefect channel estimation. This shows that the channel must be perfectly estimated; otherwise the Nash equilibrium does not become the Pareto optimum. That is, when the channel is not known perfectly, the power levels can be still determined with the game proposed in [7], since we already proved that their results are applicable to power domain NOMA, but the obtained Nash Equilibrium does not become Pareto optimum.
- The penalty and reward function parameters are locally dependent. That is, no global information is needed to determine the optimum ratio between them.

Once parameters are set to the optimum ones, the power control game can be used to find new optimum power levels that will be needed due to the highly dynamic environment, e.g., some nodes join the network while some leave. If we cannot use game theory, a power sorting algorithm in the AP must be used when the environment changes, whose complexity depends on the implementation of the algorithm as $O(N^2)$ or $O(NlogN)$ to determine the optimum power level of each node[1] and then the AP notifies each node about its optimum power level. This shows that the complexity increases with the number of nodes. On the other hand, there is no need to make any power sorting in the game theory solution, since the outcome of the power control game determines each nodes optimum power making the game theory solution more scalable.

III. Numerical Results

The aim of this section is to show the necessity of dynamic scalable power allocation policy among users, i.e., to emphasize the importance of intelligent power allocation in *uplink* NOMA. In this respect, the Pareto optimum power values found in the previous section are compared with the static power transmission strategy. Accordingly, there is a fixed total transmission power and each user determines its power level. In static approach, the power levels become independent of the channel state. The performance metric for the comparison is the BER assuming that the transmitted data is modulated with phase shift keying without any loss of generality and the total transmission power of nodes is the same for both policy in order to have a fair comparison. It is well-known that the bit error probability $P_{b,i}$ of phase shift keying modulation belonging to the i^{th} user can be expressed for a given channel and SINR by the well-known expression

$$P_{b,i} = Q(\sqrt{2SINR_i}). \tag{12}$$

A SIC receiver is employed at the AP to decode all user signals. A common impractical assumption for the SIC receivers is to make simulations with perfect symbol detection while canceling the previous users' symbols in the superposed signal. However, it is clear from (12) that there will always be bit errors for a finite SINR, i.e., perfect symbol detection is not possible. This means that the decoded i^{th} users symbols are subject to interference from the previous $(i-1)$ user's residual bit energy proportional with (12) due to bit errors or imperfect cancellation that leads to the

$$SINR_i = \frac{P_i h_i}{\sum_{j=1}^{i-1} P_j h_j + \sum_{j=i+1}^{N} P_{b,j} P_j h_j + \sigma^2} \tag{13}$$

[1]If the searching algorithm is implemented as two nested for loops, the complexity becomes $O(N^2)$. On the other hand, if linked list or other data structure is used in the searching algorithm, the complexity drops to $O(NlogN)$.

where P_i is the i^{th} user power.

Let's consider that 2 users are transmitting to a single AP at the same time and frequency with 2 different power allocation strategies. The first strategy is the Pareto optimum power levels given in (9), which can be found by the nodes as a result of the game whose penalty and reward parameters are adjusted before the game begins as in (8), and the second one is the fixed equal power policy such that both users have the same power level irrespective of the channel. Accordingly, a Rayleigh fading channel is selected. The basic assumption is that the channel is initially perfectly known by each user. Otherwise, the ratio of penalty-to-reward parameter in (8) cannot be adjusted and the Nash equilibrium does not become Pareto optimum. As shown in Fig. 2, power allocation in the *uplink* power domain NOMA has critical importance, because taking fixed equal powers disregarding the channel leads to significant degradation for the average BER. Indeed, SIC does not work efficiently without dynamic power allocation.

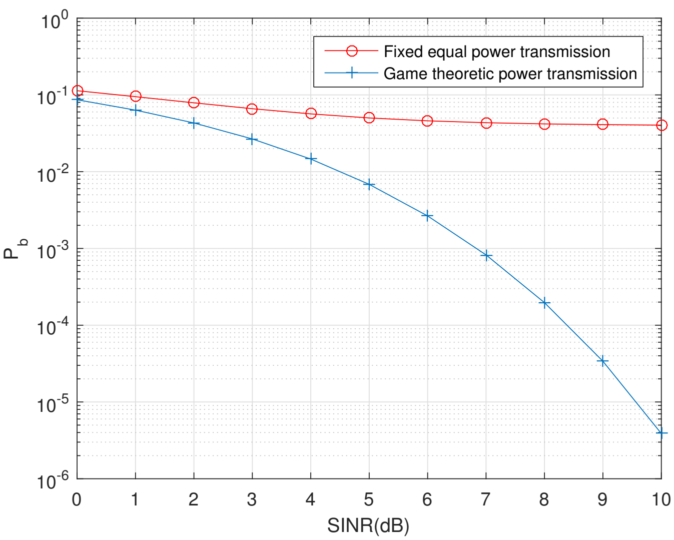

Fig. 2. Average BER comparison of 2 users for 2 different power transmission policies

The first and second users BER are related with the predetermined SINR level at the receiver. To be more specific, each users power level is adjusted so that they reach this SINR at the receiver provided that perfect SIC occurs at the receiver. However, the second user SINR and BER are affected due to the imperfect SIC of the first user bits. To observe this degradation in the BER performance of the second user, each users BER will be separately given instead of averaging. It can be observed in Fig. 3 that imperfect SIC affects the second users BER performance negligibly when their powers are allocated according to the proposed game theoretical approach assuming that each node perfectly estimates its channel.

On the other hand, the scenario for the fixed power allocation scheme such that each user has equal power, which makes the total transmission power the same with the game theoretical approach to have a fair comparison, is rather different. In this case, as shown in Fig. 4 the first user or the

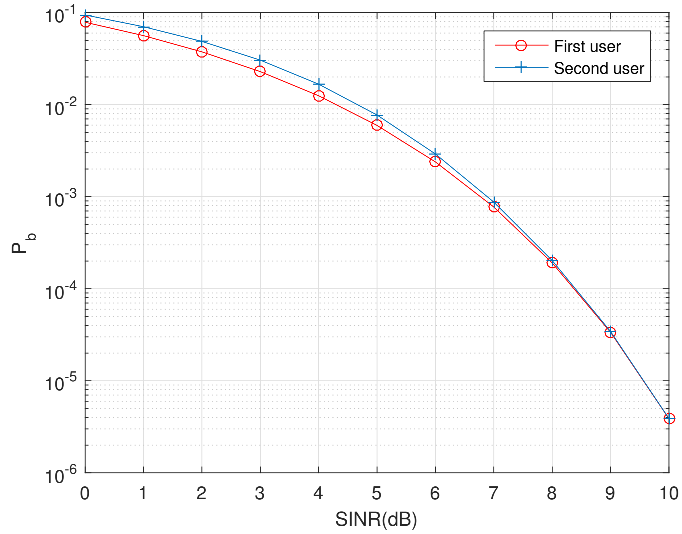

Fig. 3. Each user BER in case of imperfect SIC for game theoretic power transmission

stronger user has a poor BER performance, since its SINR at the receiver is low because the second users signal is strong as well. Note that even imperfect SIC improves the second user BER, because some bits of the first user are eliminated from the superposed signal, which increases the second user SINR.

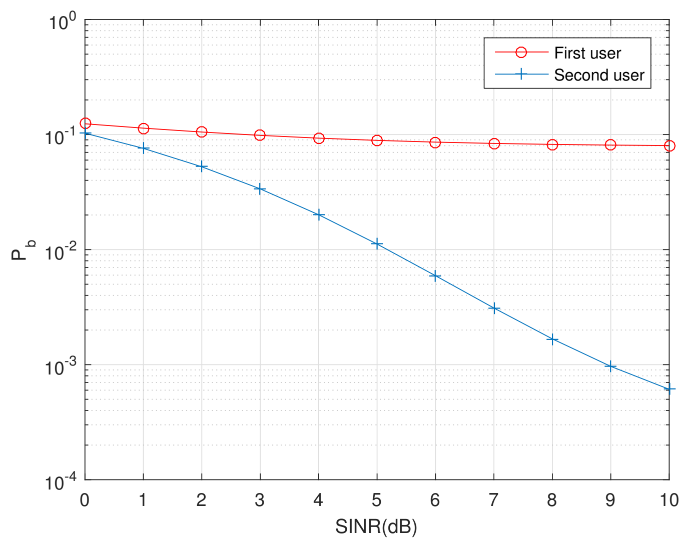

Fig. 4. Each user BER in case of imperfect SIC for fixed power transmission

To emphasize the fact that our results are independent of the number of users, the same simulation is examined when there are 3 users with Rayleigh fading channels as depicted in Fig. 5. That is, the average BER of users has considerably worse performance for fixed equal power allocation with respect to the game theoretic power allocation that optimizes the power level of each node individually. Notice that one can easily generalize these results for N nodes.

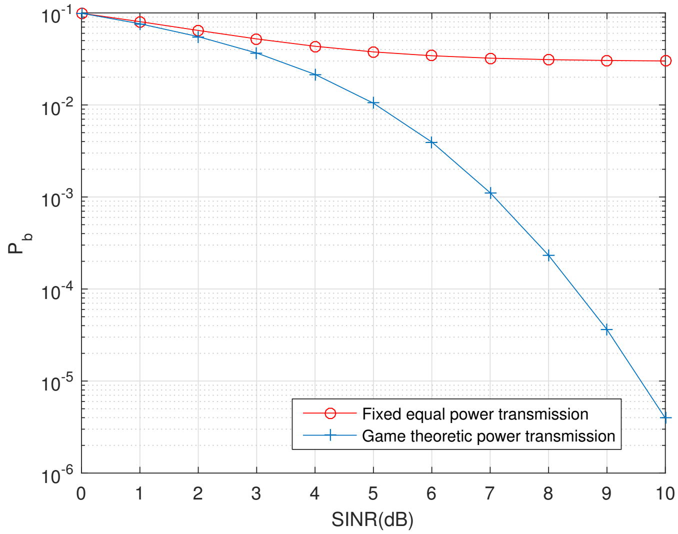

Fig. 5. Average BER comparison of 3 users for 2 different power transmission policies

These results show that game theoretical power allocation among nodes is needed to obtain a scalable and reasonable performance from power domain NOMA based on *uplink* network topology.

IV. Conclusions

One of the great challenges in the determination of power levels of NOMA is to have a scalable approach that determines the optimum levels for each user. The existing centralized solutions to ensure power allocation for *downlink* power domain NOMA cannot be applied for *uplink* power domain NOMA due to the scalability concerns. In this paper we present a scalable, distributed optimum power transmission scheme, based on game theory, for *uplink* NOMA where each user makes its own decisions on its power level. This approach will control the overall *uplink* interference among the many NOMA users. The proposed game theoretical approach uses a closed form utility function, with optimized reward and penalty function that gives the optimum power levels. Nodes can choose a utility function based upon their own observations and adjust their penalty-to-reward parameter ratio without any global knowledge. This utility function with optimized penalty-to-reward parameter ratio is very important for future power control games, since it can be used for other problems having a maximum power constraint, delay constraint, or some other constraints and can give useful results. Although one cannot claim that the optimized utility function can give the optimum results under any constraints, it can give useful results and provide intuition and guidance under a wide variety of situations.

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